QCD Evolution Workshop

Phenomenology of Sivers Effect with TMD evolution

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Summary

- Brief review of Turin parametrization with DGLAP evolution
- TMD evolution formalism (ACQR)
- Analytical approximation
- Fit of the data
- Conclusions

TMD evolution formalism*

- Phenomenological fits of the TMDs has been performed so far neglecting QCD evolution or applying QCD (DGLAP) evolution only to the collinear part of the TMD parametrizations.

- In 2011 Aybat, Collins, Qiu & Rogers presented their TMD evolution formalism applicable to the unpolarized PDFs (or FFs) and to the Sivers function

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- *J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.*
- *S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]*
- *S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]*

- Can this formalism be applied to fit the data?
- How can it be compared with the traditional approach?

Turin standard approach (DGLAP)

Turin standard approach (DGLAP)

- Unpolarized TMDs are factorized in x and $k_\perp$. Only the collinear part evolves with DGLAP evolution equation. No evolution in the transverse momenta:

$$\boxed{\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}}$$

Collinear PDF (DGLAP evolution)

Normalized Gaussian: no evolution

Turin standard approach (DGLAP)

- The Sivers function is factorized in x and $k_\perp$ and proportional to the unpolarized PDF.

$$\boxed{\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp; Q) \quad = \quad 2\mathcal{N}_q(x) h(k_\perp) \widehat{f}_{q/p}(x, k_\perp; Q)}$$

Proportional to the unpolarized TMD

$$\mathcal{N}_q(x) = N_q \, x^{\alpha_q} (1-x)^{\beta_q} \, \frac{(\alpha_q + \beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q} \, \beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e} \, \frac{k_\perp}{M_1} \, e^{-k_\perp^2/M_1^2}$$

$$\boxed{\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x, k_\perp)}$$

Turin standard approach (DGLAP)

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$$\boxed{\begin{aligned}\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) &= 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q)\\ &= 2\mathcal{N}_q(x)f_{q/p}(x;Q)\sqrt{2e}\,\frac{k_\perp}{M_1}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}\end{aligned}}$$

Collinear PDF (DGLAP)

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\,\beta_q^{\beta_q}}$$

$$\langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp) = -\frac{2k_\perp}{m_p}f_{1T}^\perp(x,k_\perp)$$

*TMD evolution formalism**

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•*S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]*
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TMD evolution formalism

➢ Let us denote with $\widetilde{F}$ either a PDF (or a FF) or the first derivative of the Sivers function in the impact parameter space:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \longrightarrow \widetilde{f}_{q/p}(x, \boldsymbol{b}_T; Q)$$

Unpolarized PDF

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \longrightarrow \widetilde{D}_{h/q}(z, \boldsymbol{b}_T; Q)$$

Unpolarized FF

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) \longrightarrow \widetilde{f}_{1T}^{\prime\perp f}(x, \boldsymbol{b}_T; Q)$$

First derivative of the Sivers function

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

Corresponding to Eq. 44 of Ref [*] with $\tilde{K}$=0 and : $\boxed{\mu^2 = \zeta_F = \zeta_D = Q^2}$

• [*]S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]

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Output function at the scale **Q**
in the impact parameter space

Input function at the scale $\mathbf{Q_0}$
in the impact parameter space

Evolution kernel

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

TMD evolution formalism

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- **Perturbative** part of the evolution kernel

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{\mathrm{d}\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

TMD evolution formalism

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$$\gamma_K(\mu)=\alpha_s(\mu)\,\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu;\frac{Q^2}{\mu^2})=\alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2}-\ln\frac{Q^2}{\mu^2}\right)$$

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0) \boxed{\widetilde{R}(Q, Q_0, b_T)} \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

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$$\widetilde{R}(Q, Q_0, b_T) \equiv \exp\left\{ \ln \frac{Q}{Q_0} \int_{Q_0}^{\mu_b} \frac{\mathrm{d}\mu'}{\mu'} \gamma_K(\mu') + \int_{Q_0}^{Q} \frac{\mathrm{d}\mu}{\mu} \gamma_F\left(\mu, \frac{Q^2}{\mu^2}\right) \right\}$$

Scale that separates the perturbative region
from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \boxed{\widetilde{R}(Q, Q_0, b_T)}\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

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$$\mu_b = \frac{C_1}{b_*(b_T)} \qquad b_*(b_T) \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}} \qquad C_1 = 2e^{-\gamma_E}$$

One of the possible prescription to separate the perturbative region from the non perturbative one

TMD evolution formalism

- At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

- **Non Perturbative** (scale independent) part of the evolution kernel that needs to be empirically modeled

$$\boxed{g_K(b_T) = \frac{1}{2} g_2\, b_T^2}$$

Common choice used in the unpolarized DY data analyses in the CSS formalism

$$g_2 = 0.68$$

Landry et al. Phys Rev D67, 073016

TMD evolution formalism

- One can get the TMD in the momentum space by Fourier transforming:

$$\widehat{f}_{q/p}(x,k_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(k_\perp b_T)\, \widetilde{f}_{q/p}(x,b_T;Q)$$

$$\widehat{D}_{h/q}(z,p_\perp;Q) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}b_T\, b_T\, J_0(\mathrm{k}_T b_T)\, \widetilde{D}_{h/q}(z,b_T;Q)$$

$$\widehat{f}_{1T}^{\perp f}(x,k_\perp;Q) = \frac{-1}{2\pi k_\perp}\int_0^\infty \mathrm{d}b_T\, b_T\, J_1(k_\perp b_T)\, \widetilde{f}'^{\perp q}_{1T}(x,b_T;Q)$$

$$\begin{aligned} f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp,\boldsymbol{S};Q) &= f_{q/p}(x,k_\perp;Q) - f_{1T}^{\perp q}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{M_p} \\ &= f_{q/p}(x,k_\perp;Q) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp;Q)\frac{\epsilon_{ij}\,k_\perp^i\,S^j}{k_\perp} \end{aligned}$$

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \boxed{\widetilde{F}(x, \boldsymbol{b}_T; Q_0)}\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

- We want to compare the effect of TMD evolution vs our traditional approach (DGLAP)

$$\downarrow$$

- Same parametrization of the input function at the initial scale in the trasverse momentum space.

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \boxed{\widetilde{F}(x, \boldsymbol{b}_T; Q_0)}\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

Example: unpolarized pdf

$$\tilde{F}(x, b_T; Q_0) = \tilde{f}_{q/p}(x, b_T; Q_0) \xrightarrow{\text{Fourier transf.}} \widehat{f}_{q/p}(x, k_\perp; Q_0)$$

$$\widehat{f}_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

Parametrization of the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$\widetilde{f}_{q/p}(x, b_T; Q_0) = f_{q/p}(x, Q_0) \exp\left\{ -\alpha^2\, b_T^2 \right\}$$

$$\widehat{f}_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$\alpha^2 = \langle k_\perp^2 \rangle / 4$$

Parametrization of the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$\widetilde{D}_{h/q}(z, b_T; Q_0) = \frac{1}{z^2} D_{h/q}(z, Q_0)\ \exp\left\{ -\beta^2\, b_T^2 \right\}$$

$$\widehat{D}_{h/q}(z, p_\perp; Q_0) = D_{h/q}(z, Q_0)\, \frac{1}{\pi \langle p_\perp^2 \rangle}\, e^{-p_\perp^2/\langle p_\perp^2 \rangle}$$

$$\beta^2 = \langle p_\perp^2 \rangle / 4z^2$$

Parametrization ot the input functions

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \boxed{\widetilde{F}(x, \boldsymbol{b}_T; Q_0)}\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$\widetilde{f}'^{\perp}_{1T}(x, b_T; Q_0) = -2\,\gamma^2\, f^{\perp}_{1T}(x; Q_0)\, b_T\, e^{-\gamma^2\, b_T^2}$$

$$\widehat{f}^{\perp}_{1T}(x, k_\perp; Q_0) = f^{\perp}_{1T}(x; Q_0)\,\frac{1}{4\,\pi\,\gamma^2}\, e^{-k_\perp^2/4\gamma^2}$$

$$4\,\gamma^2 \equiv \langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2 + \langle k_\perp^2\rangle}$$

Parametrization of the input functions

- Then the evolution equations for unpolarized TMDs become simply:

$$\widetilde{f}_{q/p}(x, b_T; Q) = f_{q/p}(x, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

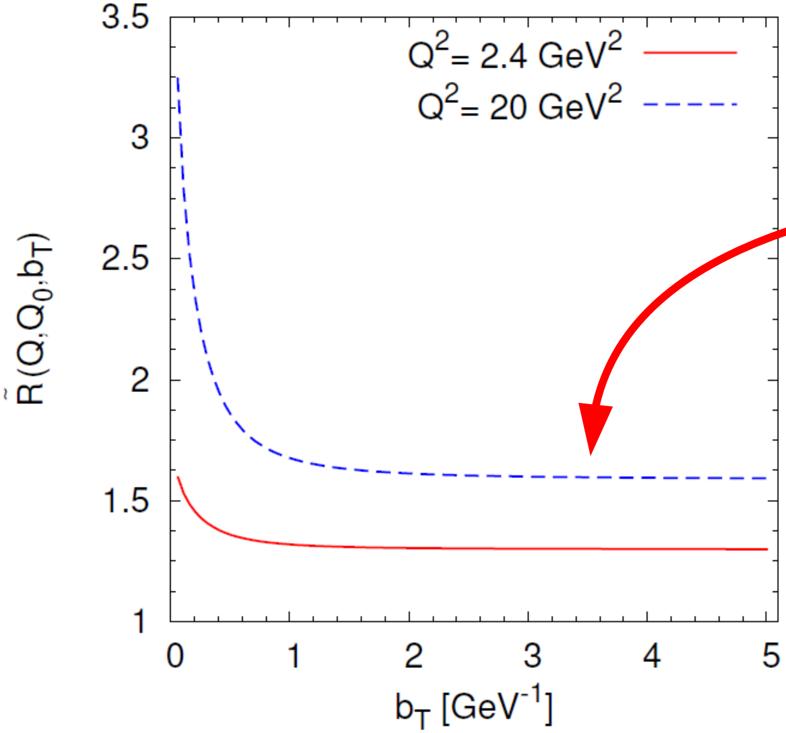
$$\widetilde{D}_{h/q}(z, b_T; Q) = \frac{1}{z^2} D_{h/q}(z, Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{-b_T^2\left(\beta^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

- While for the Sivers function we have:

$$\widetilde{f}_{1T}^{\prime\perp}(x, b_T; Q) = -2\,\gamma^2\, f_{1T}^{\perp}(x; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, b_T\ \exp\left\{-b_T^2\left(\gamma^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

Analytical (approximated) solution of the TMD evolution equation

- $\tilde{R}(Q,Q0,b_T)$ exhibits a non trivial dependence on b_T that prevents any analytical integration



$\tilde{R}(Q,Q0,b_T)$ becomes **constant** for $b_T > 1$ GeV $^{-1}$

We can therefore neglect the $\tilde{R}$ dependence on b_T and define:

$$\boxed{R(Q,Q_0) \equiv \tilde{R}(Q,Q_0,b_T \to \infty)}$$

Good approximation for large b_T i.e. small $k_\perp$

Analytical (approximated) solution of the TMD evolution equation

- For instance, replacing $\tilde{R}$ with R in the unpolarized, we get:

$$\widetilde{f}_{q/p}(x,\boldsymbol{b}_T;Q) = f_{q/p}(x,Q_0)\; R(Q,Q_0)\; \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

Which is Gaussian in b_T, and will then Fourier-transform into a Gaussian in $k_\perp$

$$\boxed{\widehat{f}_{q/p}(x,k_\perp;Q) = f_{q/p}(x,Q_0)\; R(Q,Q_0)\; \frac{e^{-k_\perp^2/w^2}}{\pi w^2}}$$

$$\boxed{w^2(Q,Q_0) = \langle k_\perp^2\rangle + 2\,g_2\ln\frac{Q}{Q_0}}$$

Analytical (approximated) solution of the TMD evolution equation

- Similarly, for the unpolarized TMD fragmentation function, we have

$$\boxed{\widehat{D}_{h/q}(z, p_\perp; Q) = D_{h/q}(z, Q_0)\; R(Q, Q_0)\; \frac{e^{-p_\perp^2/w_F^2}}{\pi w_F^2}}$$

$$\boxed{w_F^2 \equiv w_F^2(Q, Q_0) = \langle p_\perp^2 \rangle + 2z^2 g_2 \ln \frac{Q}{Q_0}}$$

Analytical (approximated) solution of the TMD evolution equation

- For the Sivers distribution function, we find:

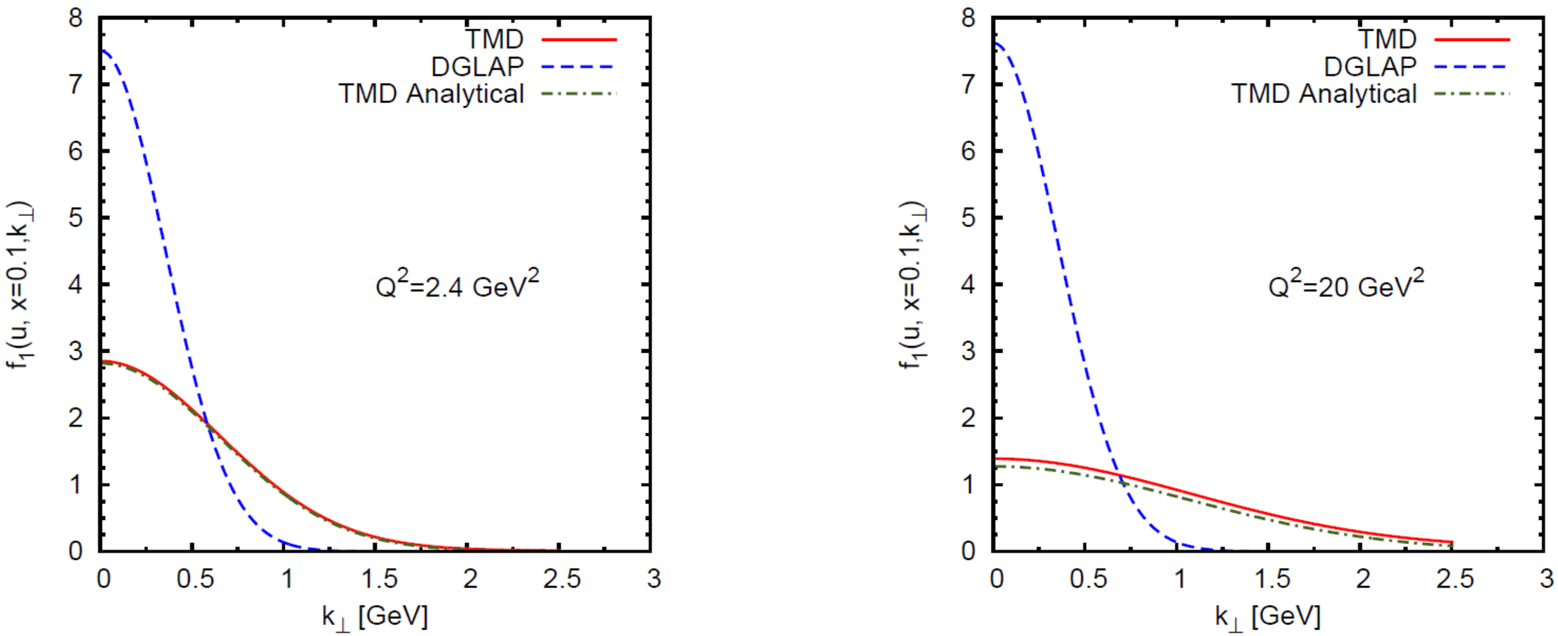
$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp; Q) = \frac{k_\perp}{M_1} \sqrt{2e} \, \frac{\langle k_\perp^2 \rangle_S^2}{\langle k_\perp^2 \rangle} \, \Delta^N f_{q/p^\uparrow}(x, Q_0) \, R(Q, Q_0) \, \frac{e^{-k_\perp^2 / w_S^2}}{\pi w_S^4}$$

$$w_S^2(Q, Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x, k_\perp) = -\frac{2k_\perp}{m_p} f_{1T}^\perp(x, k_\perp)$$

$$\langle k_\perp^2 \rangle_S = \frac{M_1^2 \, \langle k_\perp^2 \rangle}{M_1^2 + \langle k_\perp^2 \rangle}$$

Comparative analysis of TMD evoultion equations

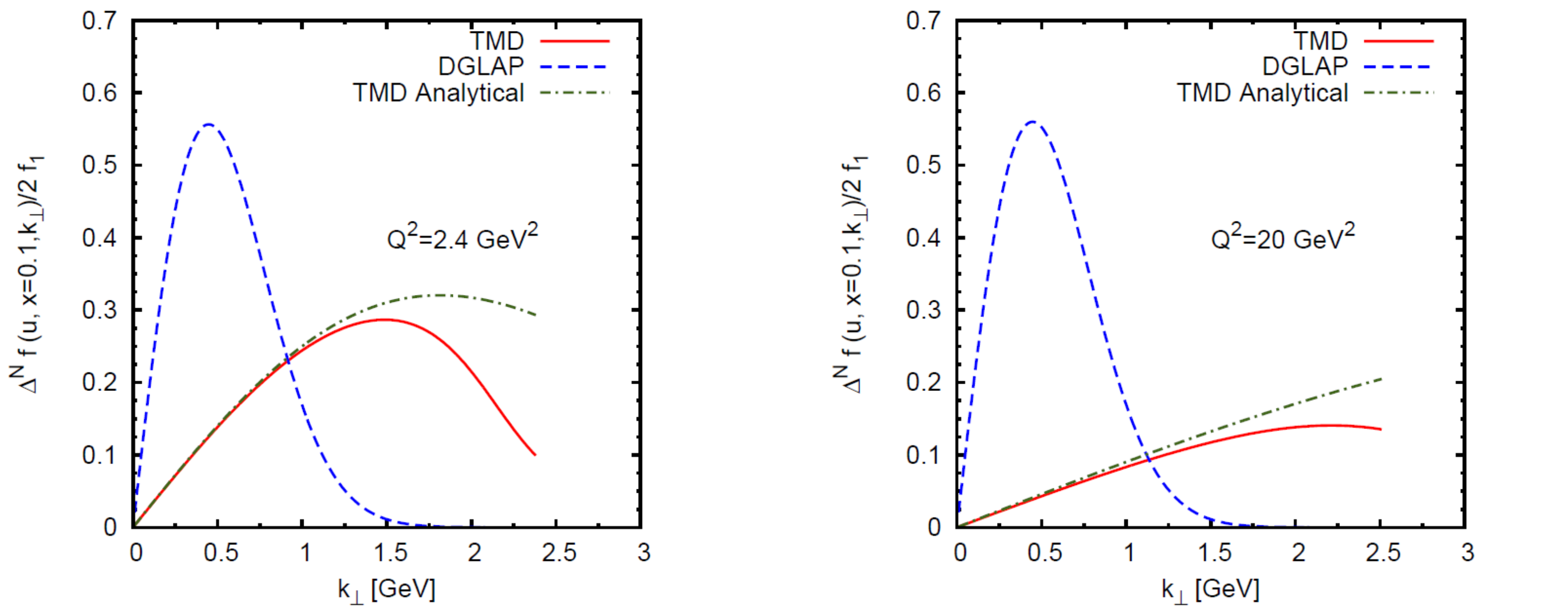


Starting scale Q_0=1 GeV
Same function at Q_0

DGLAP evolution is slow at moderate x and in this range of Q^2

For the unpolarized PDF, the analytical approximation holds up to large $k_\perp$

Comparative analysis of TMD evoultion equations



Starting scale Q_0=1 GeV
Same function at Q_0

For the Sivers function,
the analytical approximation
breaks down at large $k_\perp$ values

Fit of HERMES and COMPASS SIDIS data

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \frac{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, \Delta^N f_{q/p^\uparrow}(x,k_\perp,Q)\sin(\varphi-\phi_S)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)\sin(\phi_h-\phi_S)}{\sum_q \int d\phi_S\, d\phi_h\, d^2\boldsymbol{k}_\perp\, f_{q/p}(x,k_\perp,Q)\,\frac{d\hat\sigma^{\ell q\to\ell q}}{dQ^2}\, D_q^h(z,p_\perp,Q)}$$

$$\Delta^N \widehat{f}_{q/p^\uparrow}(x,k_\perp;Q_0) = 2\mathcal{N}_q(x)h(k_\perp)\widehat{f}_{q/p}(x,k_\perp;Q_0)$$

$$\mathcal{N}_q(x) = N_q\, x^{\alpha_q}(1-x)^{\beta_q}\,\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}$$

$$h(k_\perp) = \sqrt{2e}\,\frac{k_\perp}{M_1}\,e^{-k_\perp^2/M_1^2}$$

$$\widehat{f}_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q_0) = D_{h/q}(z,Q_0)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

11 free parameters

N_{u_v}	N_{d_v}	N_s
$N_{\bar u}$	$N_{\bar d}$	$N_{\bar s}$
α_{u_v}	α_{d_v}	α_{sea}
β	M_1 (GeV/c) .	

Fixed parameters

$$\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2$$
$$\langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$$
$$g_2 = 0.68\ \mathrm{GeV}^2$$

Fit of HERMES and COMPASS SIDIS data

- We perform 3 different fits:
  - TMD-fit (computing TMD evolution equations numerically)
  - TMD-analytical fit (solving TMD evolution equations in the analytical approx.)
  - DGLAP fit (using DGLAP evolution equation for the collinear part of the TMD)

- Data sets:
  - HERMES (2009) $\pi+$ $\pi-$ π^0 K+ K-
  - COMPASS Deuteron (2004) $\pi+$ $\pi-$ K+ K-
  - COMPASS Proton (2011) h+ h-

Fit of HERMES and COMPASS SIDIS data

χ^2 tables

11 free parameters, 261 points

TMD Evolution (Exact)	TMD Evolution (Analytical)	DGLAP Evolution
$\boxed{\chi^2_{tot} = 255.8}$	$\boxed{\chi^2_{tot} = 275.7}$	$\boxed{\chi^2_{tot} = 315.6}$
$\chi^2_{d.o.f} = 1.02$	$\chi^2_{d.o.f} = 1.10$	$\chi^2_{d.o.f} = 1.26$

Fit of HERMES and COMPASS SIDIS data

χ^2 tables 11 free parameters, 261 points

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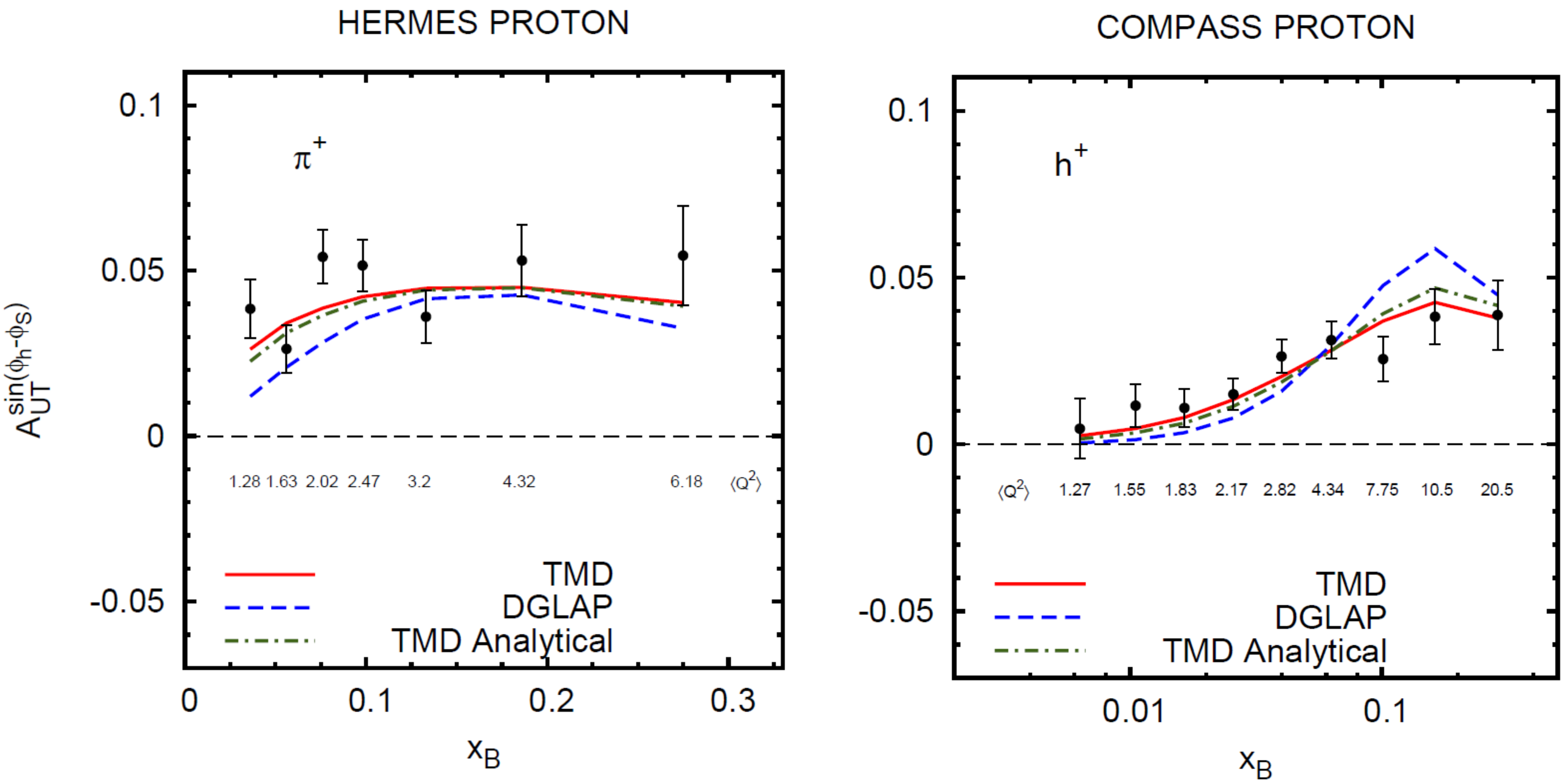
Fit of HERMES and COMPASS SIDIS data

χ^2 tables 11 free parameters, 261 points

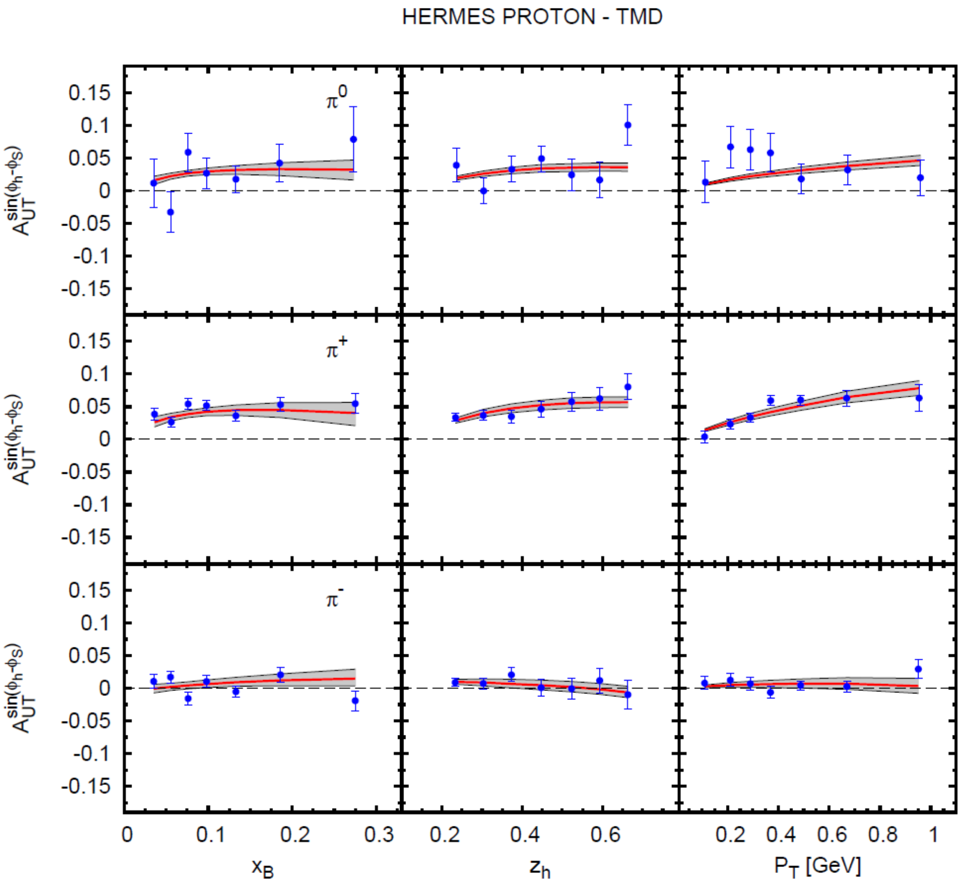
	TMD Evolution (Exact)		TMD Evolution (Analytical)	DGLAP Evolution
	$\chi^2_{tot} = 255.8$ $\chi^2_{d.o.f} = 1.02$		$\chi^2_{tot} = 275.7$ $\chi^2_{d.o.f} = 1.10$	$\chi^2_{tot} = 315.6$ $\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\boxed{\chi^2_x = 10.7}$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 9.1$	7 points	$\boxed{\chi^2_x = 12.9}$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 10.5$	$\boxed{\chi^2_x = 27.5}$ $\chi^2_z = 8.6$ $\chi^2_{P_T} = 22.5$
COMPASS h^+	$\boxed{\chi^2_x = 6.7}$ $\chi^2_z = 17.8$ $\chi^2_{P_T} = 12.4$	9 points	$\boxed{\chi^2_x = 11.2}$ $\chi^2_z = 18.5$ $\chi^2_{P_T} = 24.2$	$\boxed{\chi^2_x = 29.2}$ $\chi^2_z = 16.6$ $\chi^2_{P_T} = 11.8$

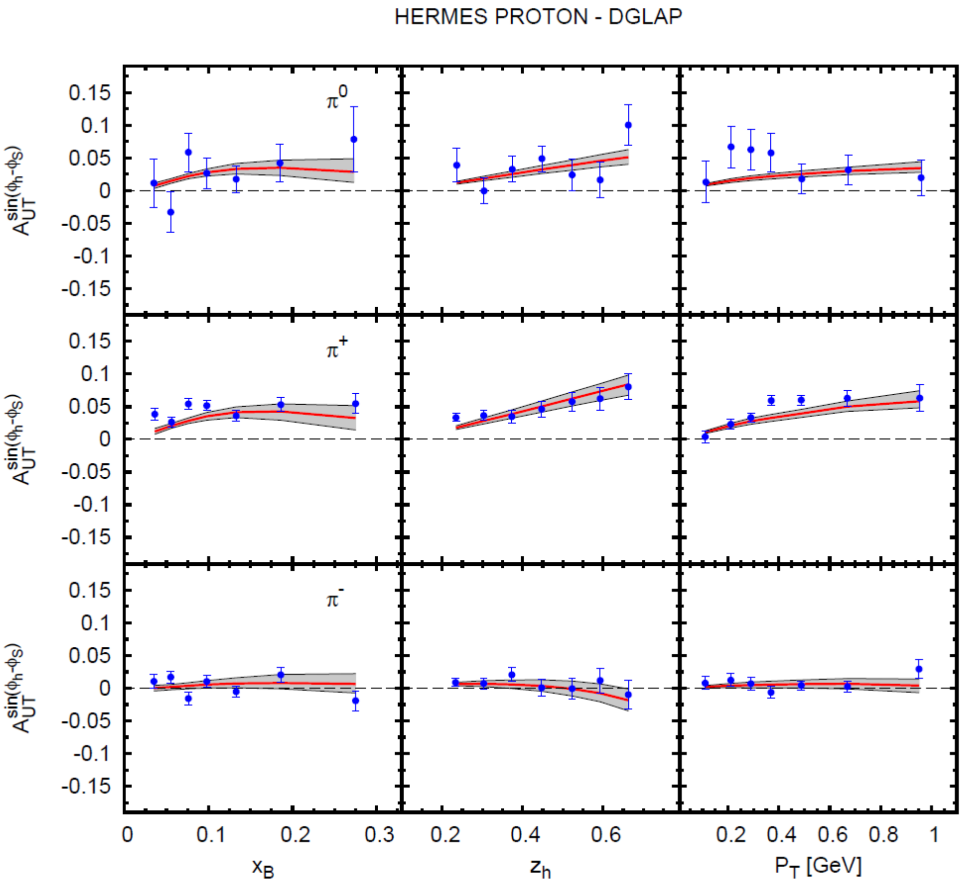
Fit of HERMES and COMPASS SIDIS data



Fit of HERMES and COMPASS SIDIS data

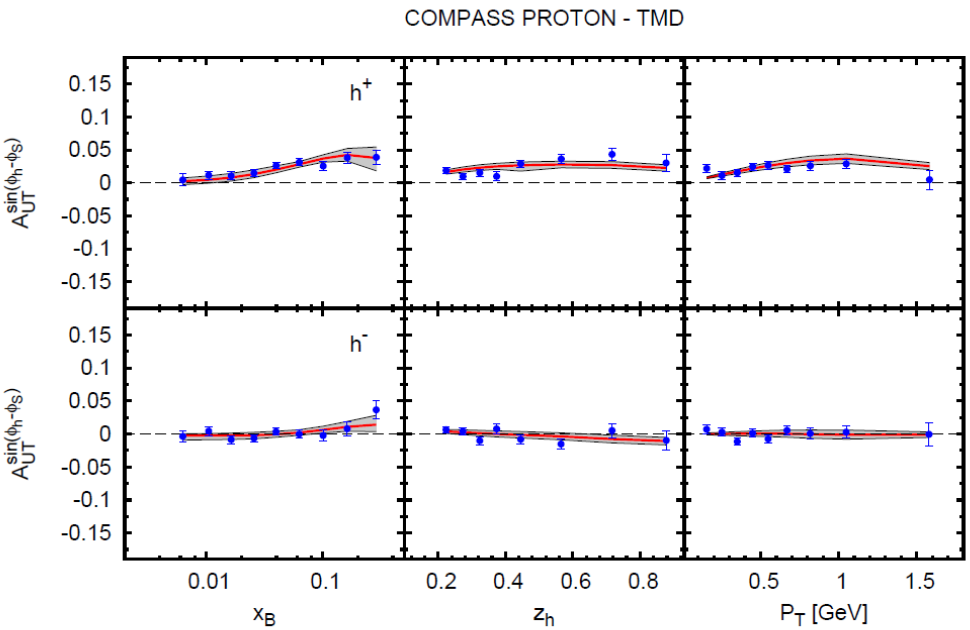
A. Airapetian et al., Phys. Rev. Lett. 103, 152002 (2009), arXiv:0906.3918 [hep-ex]

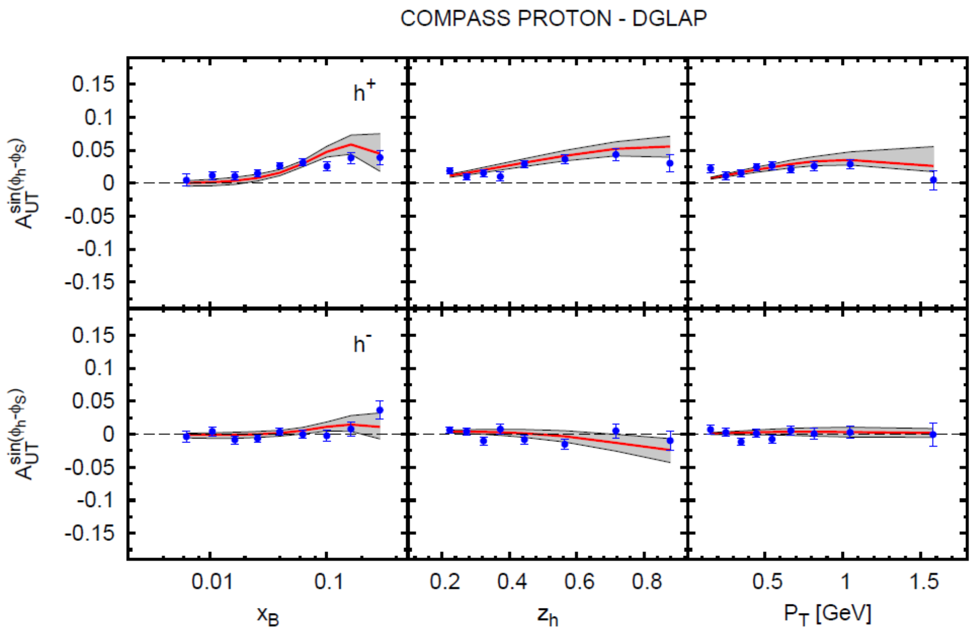




Fit of HERMES and COMPASS SIDIS data

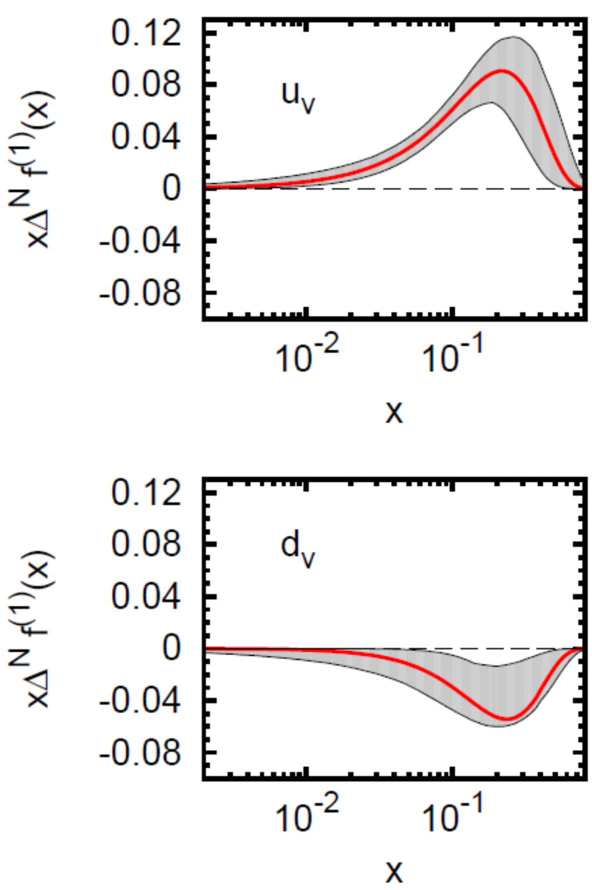
F. Bradamante, arXiv:1111.0869 [hep-ex]





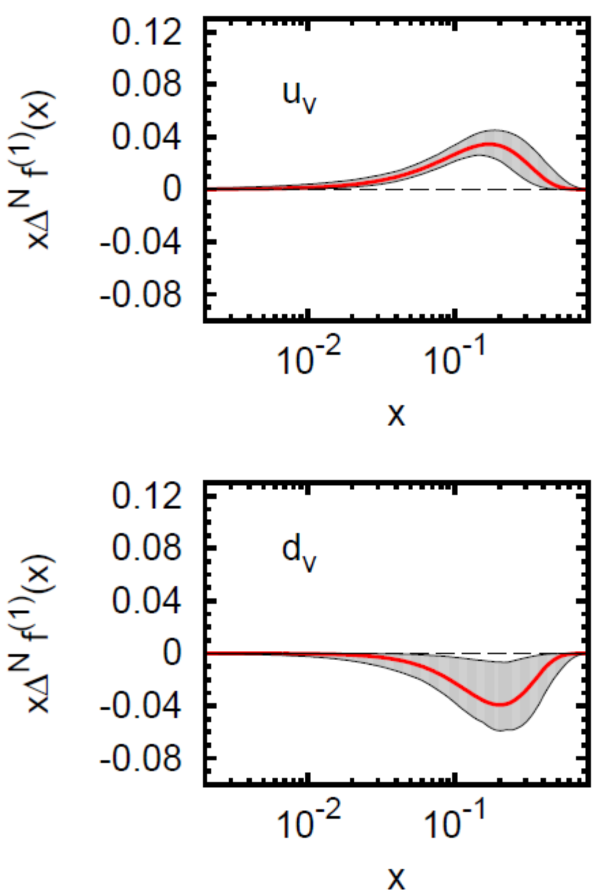
Fit of HERMES and COMPASS SIDIS data

TMD Evolution



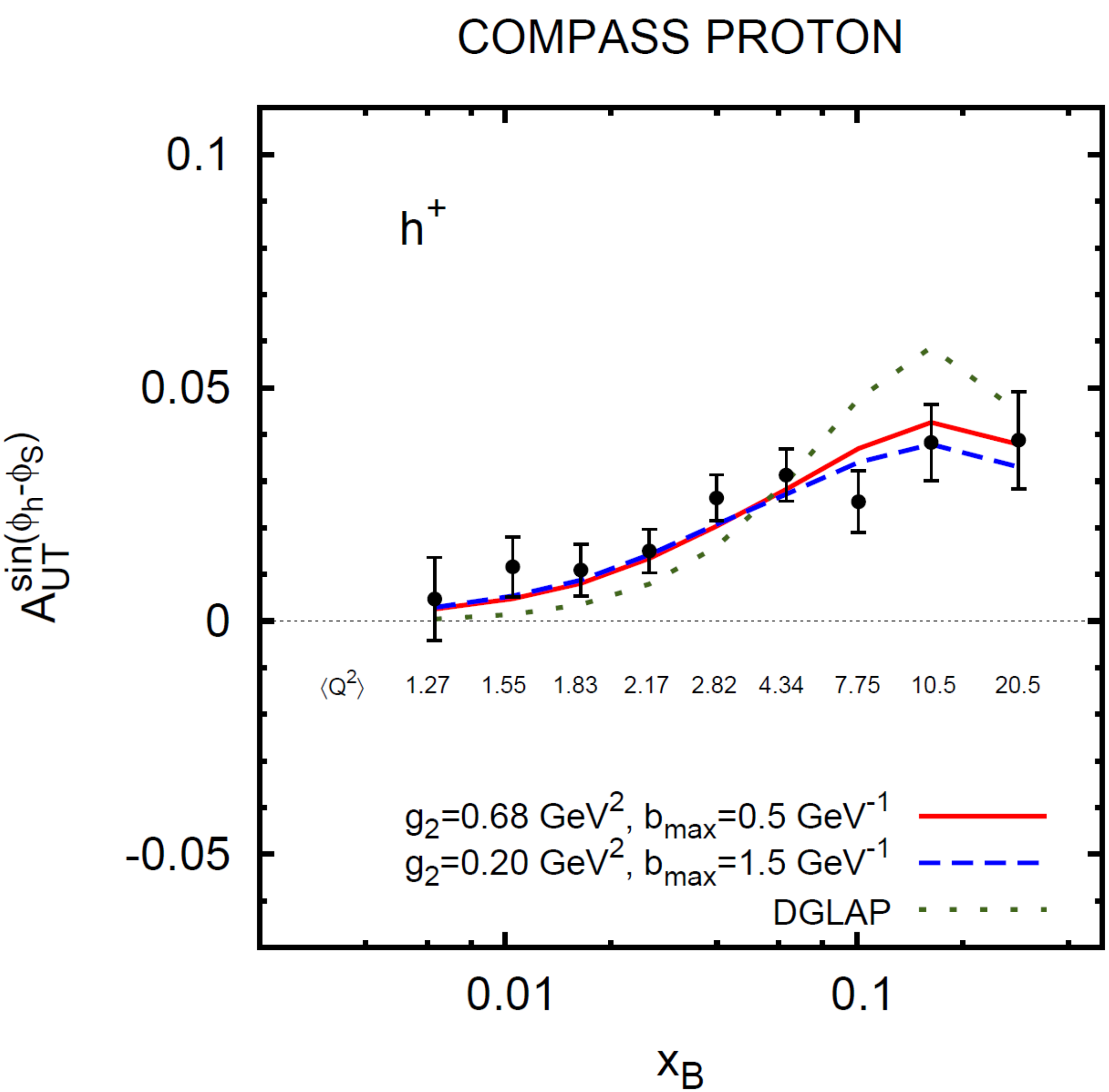
Q_0=1 GeV

DGLAP Evolution



Conclusions and further remarks

- We have analyzed the Sivers effect by up-grading old fits with the addiction of the most recent HERMES and COMPASS SIDIS data.
- We have applied **TMD evolution equations** to the phenomenological analysis of the Sivers effect.
- We have **compared** the analysis obtained using TMD evolution equations with the results found by considering the DGLAP evolution of the collinear part of the TMDs.
- **SIDIS data support the TMD evolution**, but further experimental data covering a **wider range of Q^2 values** are necessary to confirm these results.



COMPASS PROTON
h+
$A_{UT}^{\sin(\phi_h-\phi_S)}$
0.1
0.05
0
-0.05
⟨Q²⟩ 1.27 1.55 1.83 2.17 2.82 4.34 7.75 10.5 20.5
g_2=0.68 GeV², b_{max}=0.5 GeV⁻¹
g_2=0.20 GeV², b_{max}=1.5 GeV⁻¹
DGLAP
0.01
0.1
x_B

$$N_{SIDIS} \propto \Delta^N f(x,Q_0) D(z,Q_0) \sqrt{2e} \frac{P_T}{M_1} \frac{z \langle k_\perp^2 \rangle_{Siv}^2}{\langle k_\perp^2 \rangle \langle P_T^2 \rangle_{Siv}^2} e^{-P_T^2 / \langle P_T^2 \rangle_{Siv}}$$

$$\boxed{\langle P_T^2 \rangle_{Siv}^{SIDIS} = z^2 \omega_{Siv}^2 + \omega_{FF}^2}$$

$$\boxed{w_S^2(Q,Q_0) = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0}}$$

$$\boxed{w_F^2 \equiv w_F^2(Q,Q_0) = \langle p_\perp^2 \rangle + 2 z^2 g_2 \ln \frac{Q}{Q_0}}$$

- Here is squared, strongly suppress the asymmetry as it becames larger and larger

- 0.2 <z<0.8

- Numerator of the asymmetry in analytical approximation for a DY process

$$N_{DY} \propto \Delta^N f(x_1,Q_0) f(x_2,Q_0) \sqrt{2e}\,\frac{P_T}{M_1}\,\frac{\langle k_\perp^2\rangle_{Siv}^2}{\langle k_{\perp 1}^2\rangle \langle P_T^2\rangle_{Siv}^2}\, e^{-P_T^2/\langle P_T^2\rangle_{Siv}}$$

$$\boxed{\langle P_T^2\rangle_{Siv}^{DY} = \omega_{Siv}^2 + \omega_2^2}$$

$$\boxed{w_S^2(Q,Q_0) = \langle k_\perp^2\rangle_S + 2 g_2 \ln\frac{Q}{Q_0}}$$

$$\boxed{w^2(Q,Q_0) = \langle k_\perp^2\rangle + 2\, g_2 \ln\frac{Q}{Q_0}}$$

- Here is squared, strongly suppress the asymmetry as it becames larger and larger

- g_2 is more crucial for DY processes than for the present SIDIS data (because of a wider kinematical range in Q^2)

$$\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) = \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu_0, Q_0^2) \exp\Bigg\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \Bigg\}. \quad (44)$$